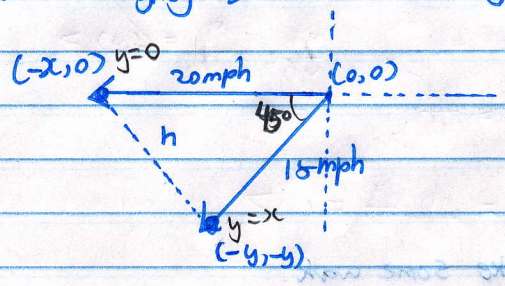
① Two boats leave a port at the same time, one travelling west at 20 mph and the other travelling southwest at 15 mph. At what rate is the distance between them changing 30 min after they leave the port?



Ans: Let the coordinates of the first boat = $(-x, 0)$

" " " " second boat = $(-y, -y)$

Distance between the two boats = Distance between $(-x,0)$ and $(-y,-y)$

$$D^2 = (y^2 - x^2) + y^2$$

Also have $\sin 45° = \frac{y}{h} = \frac{1}{\sqrt{2}}$

$$h = \sqrt{2} \times y$$

$D^2 = (y^2 - x^2) + y^2$, get $2D \cdot \frac{dD}{dt} = 2(y-x)\left(\frac{dy}{dt} - \frac{dx}{dt}\right) + 2y \cdot \frac{dy}{dt}$ (Chain Rule)

Given: $\frac{dx}{dt} = 20$ mph

$$\frac{dy}{dt} = \frac{1}{\sqrt{2}} \cdot \frac{dr}{dt} = \frac{15}{\sqrt{2}} \approx 10.6 \text{mph}$$

After 30 mins, we have $x = 10$ and $y = \frac{15 \cdot \sqrt{2}}{4}$ (After 30min $y = \frac{15}{2\sqrt{2}} = \frac{15\sqrt{2}}{4}$)

$$D^2 = (y^2 - x^2) + y^2 = \left(\left(\frac{15\sqrt{2}}{4}\right)^2 - (100)\right) + \left(\frac{15\sqrt{2}}{4}\right)^2$$

$$\Rightarrow D \approx 11.04 \text{ miles.}$$

Substituting D, y, x, $\frac{dy}{dt}$ and $\frac{dx}{dt}$ on $2D\frac{dD}{dt} = 2(y-x)\left(\frac{dy}{dt} - \frac{dx}{dt}\right) - 2y\frac{dy}{dt}$,

we get $\frac{dD}{dt} \approx 14.1$ mph